

# Interaction of transfer processes during unsteady evaporation of water droplets

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## Abstract

The change of water droplets state is modelled numerically under various heat and mass transfer conditions during their unsteady evaporation. The modelling is performed using the method of combined analytic–numeric research of heat and mass transfer in a two-phase “droplets–gas” flow. The algorithm of an iterative research is constructed for the analytically obtained system of integral equations. Regularities of heat transfer process interaction are examined. The dependence of the droplet state change on its heating manner is determined. Unsteadiness and interaction of transfer processes, as well as selectivity of radiant absorption in water droplets are evaluated. It is indicated that cognition of the droplet state change regularities in the case of conductive heating is very important in determination of two-phase flow and in construction of an engineering research method.

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## 1. Introduction

Two-phase flows consisting of droplets and gas are effective means of equipment protection against direct influence of high temperature. Heat and mass transfer processes, which take place in a water droplet, are important in air conditioning systems—air drying or humidification in evaporation chambers; in cooling of high temperature gas flows; fire extinguishing, etc. Spray of water into flame or into air supply system of an internal combustion engine is effective in regulation of maximum combustion temperature, optimization of combustion process or reduction of  $\text{NO}_x$  emission. Recently scientists show their interest in the influence of water injection into gas turbines on transfer processes during gas compression and expansion. A water droplet relates all aforementioned cases. The rate of a droplet heating and evaporation is the essential factor of

the state change of a two-phase system “droplets–gas”. The intensity of the transfer processes in droplets is decided by the peculiarities of heat transfer interaction in a droplet and heat and mass transfer interaction in surrounding gas. Unsteadiness, intensive interaction and radiant intensity are the characteristic features of heat and mass transfer processes in the two-phase flows, consisting of gas and droplets.

Droplet researches have been carried out for more than a century [1]. They cover such wide spectrum of heat and mass transfer problems that it is impossible to overview all research methods in this study, even very briefly. In early studies the main attention was paid to the research of external heat and mass transfer of stagnant pure liquid droplets in low temperature surroundings. When researching equilibrium transfer processes, it was enough to determine heating intensity of evaporating droplets. In later researches the influence of more complicated boundary conditions of heat and mass transfer on the intensity of droplet heating and phase transformation was taken into account. The necessity of evaluation of heat and mass transfer process unsteadiness showed the importance of the internal droplet problem, because in this case it was

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**Nomenclature**

$a$	thermal diffusivity	$\varphi$	angle between the opposite direction of the normal to the surface element and the incident beam
$B$	Spalding transfer number	$\lambda$	thermal conductivity
$c$	speed of light	$\Theta$	additional function, used in Eq. (24)
$c_p$	mass specific heat	$\mu$	molecular mass
$C_1$	droplet drag coefficient	$\rho$	density
$D$	mass diffusivity	$\tau$	time, optical thickness
$Fo$	Fourier number	$\omega$	wave number
$G$	mass flow rate	$\psi$	azimuthal angle
$I$	control time index		
$I_\omega$	intensity of radiation		
$k_c$	effective conductivity parameter		
$k_\omega$	spectral index of absorption		
$L$	latent heat of evaporation		
$m$	vapour mass flux		
$n$	number of the term in infinite sum		
$n_\omega$	spectral index of light refraction		
$N$	number of terms in infinite sum		
$Nu$	Nusselt number		
$p$	pressure		
$Pe$	Peclet number		
$Pr$	Prandtl number		
$q$	heat flux		
$r$	coordinate of a droplet		
$R$	radius of a droplet		
$Re$	Reynolds number		
$R_\mu$	universal gas constant		
$s$	freely selected direction		
$T$	temperature		
$w$	velocity		
$\Delta w$	slip velocity of a droplet in gas		
<i>Greek symbols</i>			
$\beta, \gamma$	angles, estimating the peculiarities of spherical geometry, when calculating radiation		
$\chi_\omega$	spectral radiant absorption		
$\eta$	non-dimensional coordinate		
<i>Subscripts</i>			
c	convective		
C	droplet centre		
e	equilibrium evaporation		
f	evaporation		
g	gas		
gv	gas–vapor mixture		
$i$	time index in a digital scheme		
$I$	index of control time		
$j$	index of droplet cross-section		
$J$	index of droplet surface		
k	conductive		
m	mass average		
l	liquid		
r	radiation		
R	droplet surface		
v	vapor		
$\omega$	spectral		
$\Sigma$	total		
0	initial state		
$\infty$	far from a droplet		
<i>Superscripts</i>			
+	external side of a surface		
–	internal side of a surface		

necessary to determine the distribution of heat flow for droplet heating and evaporation. When modelling heat transfer in evaporating droplets, often assumptions are made, which allow avoiding the necessity to evaluate the interaction of thermal radiation and convection processes. History of an internal droplet modelling is thoroughly discussed in [2], where essential results are presented in the case, when radiation influence on the droplet state can be neglected.

According to their size all droplets can be classified into three groups [3]: large, when  $Kn < 0.01$ ; medium, when  $0.01 < Kn < 0.3$ ; and small, when  $Kn > 0.3$ . The influence of radiant absorption intensity in semitransparent liquid on the interaction of transfer processes is important in large droplets [4]. The influence of the Stefan hydrodynamic flow is essential in gas–liquid vapour mixture that

surrounds the droplet [5]. The impact of radiant flow on convective heat flow in gas is not significant [6]; therefore these flows in gas can be calculated independently, neglecting their interaction. The intensity of convective heating of an evaporating droplet is usually calculated according to the similarity theory principles for a solid sphere, while the influence of phase transformations is evaluated using the functions of the Spalding transfer number [2,5,7], etc. Logarithmic and power expressions of these functions are used mostly often. Radiation makes a direct influence on the intensity of conductive heat flow in a non-isothermal droplet, since it changes the local gradient of the droplet temperature field. In its turn, the temperature field, altered due to the interaction of heat transfer processes, evokes the changes of liquid spectral complex index of refraction. Therefore the conditions of light ray propagation in a

droplet and optical effects on the droplet surface change as well. Radiant flow is practically not absorbed on the surface of semitransparent liquid; therefore it has no direct influence on the droplet surface temperature. The influence of radiant flow on the droplet surface temperature evidences itself through the altered gradient of the droplet temperature field. The change of energy flow intensity changes the rate of droplet heating and evaporation, which actually determines the influence of the Stefan hydrodynamic flow on convective heating intensity of the evaporating droplet. Vapour flow, which spreads from the droplet surface, resists an inflow of hot gas towards the droplet. That increases the thickness of thermal boundary layer [5] and the intensity of convective heating to the droplet decreases, if compared to convective heating of a solid sphere. The Stefan hydrodynamic flow increases droplet evaporation intensity.

Archimedean forces, which arise in a non-isothermal droplet under the conditions of usual thermal processes, do not evoke liquid circulation [8–10]. Though friction drag forces, arising due to the difference between droplet and gas velocities, evoke liquid circulation in the droplet. Then liquid circulation is evaluated by the effective thermal conductivity  $\lambda_{\text{ef}} = \lambda_1 \cdot k_c$ . The effective conductivity parameter  $k_c$  is the function of Peclet number and its value begins to differ perceptibly from 1, as  $Pe_1 > 5$  [7]. Such method of the internal droplet problem solution allows evaluating the influence of heat transfer with circulating liquid in the energetic balance of the droplet. Though it is not possible to evaluate the status of every droplet cross-section.

Radiation effects are significant in high temperature two-phase flows. It is one of the most complicated problems of two-phase fluid mechanics. The possibilities of its numerical investigation were expanded by the rapid progress of computational technique. Volumetric radiation absorption in droplets using geometric optics approach was examined in the study [11]. It was assumed that absolutely black body radiation acts on liquid from its surroundings. Radiation absorption in water, *n*-heptane and *n*-decane droplets was calculated using the electromagnetic theory method in [12]. In the study [13] the influence of radiation absorption on a droplet heating was examined under the condition that the droplet is an isotropic homogeneous sphere, though neglecting radiation emission in the droplet itself. The results of the numeric research affirm radiation influence on droplet heating and evaporation processes. Local radiant flux in liquid using light ray tracing method was examined in [14]. In this study radiation absorption was modelled in semitransparent liquid sphere, slab and cylinder, as they are uniformly acted by isotropic radiant flux. The influence of a volume shape on radiation absorption process was indicated. External radiation absorption in a dispersed system of spherical volumes is modelled in [15]. Asymmetric illumination of separate particles is approximately equated to a uniform illumination from two hemispherical volumes, oriented according to the direction of spectral radiation flow. Calculations were

carried out using Mie radiation theory. The integral equation of local radiant flux in a semitransparent sphere is constructed in the study [16]. For that spectral optical density along freely chosen direction was replaced by optical density along the direction of a semitransparent sphere dimension. The advantage of this method is that liquid radiation emission and dependence of optical spectral properties on liquid temperature is evaluated. Evaluation of spectral optical properties of a semitransparent sphere is very important, and that is verified in all aforementioned studies. The influence of liquid temperature on energy transfer in water droplets is researched in [17]. Spectral absorption and reflection coefficients, as well as total internal reflection angles in wide range of boundary conditions were calculated in this study. The research results showed that the change of optical properties influences local radiant flux in warming water droplets.

Radiation absorption can significantly distort steady temperature field in a semitransparent liquid droplet and even change the direction of conductive component of the total heat flux [18]. The influence of unsteady radiant–conductive transfer process interaction in initially isothermal droplet is thoroughly examined in [4,10,19]. The conception of a complex droplet problem is presented; close interrelation between internal and external droplet problems is indicated. According to the place of temperature field maximum of a droplet it is suggested to classify the change of a droplet state into the initial, transient and final periods. Transfer process interaction is peculiar in those periods. Though liquid circulation influence is neglected in these studies and the numeric scheme requires iterative calculations, which are CPU time consuming. That limits application of analytical–numeric research method of the droplet internal problem in modelling of transfer process interaction in two-phase flows “gas–droplets”. An important step towards liquid circulation evaluation was made in the studies [20,21]. They contain a developed method of unsteady radiant–conductive heat transfer problem solution (presented in [4]) and indicate a possibility to use time-intermittent numeric research scheme.

Heating and evaporation of water droplets under the influence of monochromatic radiation is thoroughly examined in [22,23]. Fast heating and slow heating modes of droplets were determined. When the intensity of external radiation is high, large droplet warms very quickly without any sensible influence of conduction. In such case temperature profiles repeat radiant flux profiles in a droplet. Energetic equilibrium on the droplet surface has not been stabilized yet, though non-intensive droplet phase transformations quickly shift to an explosive mode of droplet evaporation. In this case external energy source dominates, and it is possible to neglect droplet evaporation. During slow heating mode the influence of thermal conduction and evaporation processes is sensible. In this case a widely used condition of quasi-steady state of a droplet is valid, on which the existence of energy flux equilibrium on a droplet

surface is based. Under conditions of asymmetric radiation temperature fields inside a droplet are close to spherically symmetric, which are characteristic to a uniform source case. Maximal temperature of a droplet centre is higher than liquid saturation temperature only in a few degrees, i.e., explosive evaporation mode is not reached.

Application of the results of fundamental two-phase flow research in various CFD (Computational Fluid Dynamics) codes is limited by strict requirements for a droplet problem solution time. Therefore some difficulties of the droplet internal problem simplification and the reliability of obtained results might arise. Heating and evaporation of fuel droplets using a parabolic temperature profile model was analyzed in [24,25]. Advanced D<sup>2</sup> law model evaluates heating of a droplet. This form of the D<sup>2</sup> law is called the ellipse law and is based on elliptical presentation of the droplet diameter variation in time. Actually it is one more approach, suitable to be used in CFD codes. It does not require a lot of CPU time, though it does not allow evaluating the interaction of transfer processes.

Droplet research is quite urgent in our days due to wide technological application of two-phase droplet–gas flows and the necessity to know better the regularities of unsteady heat and mass transfer interaction. This study contains modelling of heat transfer interaction in large water droplets during their unsteady evaporation process.

## 2. Problem formulation

Unsteady and equilibrium phase transformation modes of a droplet can be distinguished according to the distribution of heat supplied to the droplet from outside between the energy flux  $q_h^-$  that heats the droplet and the energy flux  $q_f^+$  that evaporates it. An equilibrium evaporation mode is understood as the case, when all heat supplied to the droplet from outside is used for its evaporation. The heat flux, which heats the droplet, by its meaning corresponds to the total heat flux on the internal side of the droplet surface, i.e.  $q_h^- \equiv q_{\Sigma}^-$ . In the case of equilibrium droplet evaporation  $q_{\Sigma}^- = 0$ , as droplet temperature remains constant. As the droplet cools, the total heat flux in it indicates the input of the internal droplet energy to the process of equilibrium evaporation:

$$q_{\Sigma,c}^- \equiv \frac{1}{3} \rho_l c_{p,l} R \frac{dT_{l,m}}{d\tau}. \quad (1)$$

The intensity of droplet phase transformation energy is proportional to liquid vapour flux on the droplet surface. By its meaning it corresponds to the difference of total heat fluxes on the droplet surface:

$$q_f^+ = m^+ L \equiv q_{\Sigma}^+ - q_{\Sigma}^-. \quad (2)$$

Liquid vapour mass flux on the droplet surface depends on diffusive and convective vapour fluxes in gas and vapour mixture near the droplet. It is directly proportional to liquid vapour mass diffusivity; inversely proportional to

the droplet size and temperature of gas–liquid vapour mixture near the droplet surface. Liquid vapour mass flux can be defined using the logarithmic dependence of partial vapour pressure [26,27]:

$$m^+ = \frac{D\mu_v}{T_{R,vg} R_{\mu} R} p \ln \frac{p - p_{v,\infty}}{p - p_{v,R}}. \quad (3)$$

In the case of large droplets temperature of liquid vapor and gas mixture near the droplet surface corresponds to temperature of the evaporating droplet surface: ( $T_{R,vg} \equiv T_R$ ); in the case of medium size droplets these temperatures are close to one another [28].

Droplet surface temperature is decided by the interaction of energy fluxes on its surface. This temperature is calculated by numerically solving the following equation:

$$q_c^+(\tau) + [q_r^+(\tau) - q_r^-(\tau)] - q_c^-(\tau) - q_f^+(\tau) = 0. \quad (4)$$

The second term of the non-linear integral and differential equation (4) represents energy of the intervals of radiation spectrum, in which spectral index of absorption of light in liquid is very high. A droplet absorbs this energy by its surface. The influence of the second term of Eq. (4) on the dynamics of semitransparent liquid droplet surface temperature  $T_R(\tau)$  is small.

Droplet evaporation dynamics depends on liquid evaporation rate:

$$\frac{\partial(\rho_l V_l)}{\partial\tau} = -4\pi R^2 m^+. \quad (5)$$

The intensity of external convective heating depends on the dynamics of a droplet slip in gas flow:

$$\frac{dw_l}{d\tau} = \frac{3}{8} \frac{C_1}{R} \frac{\rho_g}{\rho_l} \frac{|w_l - w_g|}{(w_l - w_g)^{-1}}. \quad (6)$$

The friction drag coefficient of an evaporating droplet [29]:

$$C_1 = \frac{24 + 4.8Re^{0.63}}{(1+B)^{0.2} Re}. \quad (7)$$

The function of the Spalding transfer number  $B$  in the expression (7) [11]

$$B = \frac{c_{p,g}(T_g - T_R)}{L + q/m^+} \quad (8)$$

evaluates the influence of phase transformations on the friction drag coefficient of an evaporating droplet.

Convective heat flux to an evaporating droplet [5]:

$$q_c^+ = f_B^{-0.7} \cdot \frac{\lambda_{vg}(2 + 0.57Re^{1/2} Pr^{1/3})}{2R} \cdot (T_g - T_R). \quad (9)$$

The expression (8) contains heat flux  $q$ , which during unsteady droplet evaporation is proportional to the gradient of temperature field in the droplet [10]:

$$f_B = 1 + \frac{c_{p,vg}(T_g - T_R)}{L} \left[ 1 + \frac{\lambda_1 \left. \frac{\partial T(r,\tau)}{\partial r} \right|_{r=R}}{q_c^+} \right]. \quad (10)$$

If liquid is circulating in the evaporating droplet, thermal conductivity  $\lambda_l$  in Eq. (10) is replaced by effective thermal conductivity  $\lambda_{\text{ef}}$ . The effective conductivity parameter is calculated by the value of Peclet number [7]:

$$k_c = 1.86 + 0.86 \tanh[2.245 \log_{10}(Pe_1/30)]. \quad (11)$$

In order to calculate the unsteady temperature field in a droplet, an internal droplet problem is being solved. In the case of interacting radiant–conductive heat transfer in a semitransparent liquid droplet an internal droplet problem is defined by the following system of integral and differential equations:

$$\rho_l c_{p,l} \frac{\partial T(r, \tau)}{\partial \tau} = \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \left[ \lambda_l \frac{\partial T(r, \tau)}{\partial r} + q_r(r, \tau) \right] \right\}, \quad (12)$$

$$\left. \frac{\partial T(r, \tau)}{\partial r} \right|_{r=0} = 0, \quad (13)$$

$$q_r(s) = \int_0^\infty \int_0^{2\pi} \int_0^\pi I_\omega(s) \sin \varphi \cos \varphi \, d\varphi \, d\psi \, d\omega, \quad (14)$$

$$\frac{1}{c} \frac{\partial I_\omega(s)}{\partial \tau} + \frac{\partial I_\omega(s)}{\partial s} = \chi_\omega(s) [n_\omega^2(s) I_{\omega 0}(s) - I_\omega(s)], \quad (15)$$

$$\bar{n}_\omega = n_\omega - ik_\omega. \quad (16)$$

Energy equation (12) defines temporal and spatial change of unsteady temperature field of a droplet. This change is evoked by the variation of conductive and radiant local heat fluxes. The condition (13) distinguishes the case of symmetric heat transfer. The integral equation (14) defines local radiant heat flux in a droplet, which depends on spectral radiant intensity, described by the expression (15). The latter depends on the intensity of light wave absorption in liquid, which is determined by a liquid spectral index of absorption, defined by the expression (16). The real part of the complex spectral index of light refraction  $\bar{n}_\omega$  corresponds to the spectral index of reflection; the imaginary part corresponds to the spectral index of absorption. Physics of interacting heat transfer processes and their effects on each other is particular. Interacting heat transfer processes deform droplet temperature field and change its gradients. That evokes direct change of conductive local heat flux. Though the influence of the transfer process interaction on radiant heat flux is indirect. This influence evidences as the change of liquid optical spectral properties due to changed liquid temperature. In this case the intensity of spectral radiation absorption in a droplet is changed; the conditions of light reflection on the droplet surface and light ray refraction between gas and liquid are changed as well.

Evaluating Eq. (14), which define radiant flux, the energy equation (12) can be defined as a non-linear second-degree parabolic type differential–integral equation. In general case its analytical solution is not possible. Note that a direct numeric solution of Eqs. (12)–(16) using the method of finite differences is also highly complicated—difficulties arise due to calculation of radiant flux divergence in Eq. (12), indeterminacy of the selection of time and space coordinate step and selectivity of spectral radiation. The method of combined analytic–numeric research of

the system (12)–(16) is used in this study. This method is based on experience, gained in [4,10] studies. The essence of this method is the following: the system of Eqs. (12)–(16) is transformed into the system of integral equations and the scheme of iterative numeric calculations for a new system is constructed. The convergence of such scheme of iterative numerical research would be easy to control. Eq. (4) is the boundary condition of the droplet internal and external problems. It corresponds to the third-degree boundary conditions. Therefore the unambiguously conditions for the system of Eqs. (12)–(16) are formed at the beginning of the research in regards to yet unknown droplet surface temperature and spectral radiation intensity in the droplet:

$$\begin{aligned} T(r, 0) &= T_{R,0}(r), \quad R(0) = R_0, \\ T(R, \tau) &= T_R(\tau), \quad I_\omega(R, \tau) = I_{\omega,R}(\tau). \end{aligned} \quad (17)$$

The premise of the iterative numeric research scheme allows solving the energy equation (12) and radiant flux Eq. (13) independently: when solving Eq. (12), radiant flux is set to be a known parameter; when solving Eq. (13)—temperature field is set to be defined.

It is possible to state that the first member of Eq. (15), which defines temporal change of spectral radiant intensity, is infinitesimal, because light propagation speed is far and away higher than thermal conduction propagation speed. Then the change of light intensity along freely chosen direction in a droplet  $s$  is easily linked to its change along the droplet coordinate  $r$  [16]:

$$\frac{dI_\omega}{ds} = \pm \frac{\sqrt{r^2 - R^2 \sin^2 \beta}}{r} \frac{dI_\omega}{dr}. \quad (18)$$

By using symbolic denotation of optical thickness

$$\tau_{r_{j-1}}^{r_j} \equiv \int_{r_{j-1}}^{r_j} d\tau_{r_{j-1}}^{r_j} = \frac{\chi_\omega dr'}{\sqrt{1 - (r/r')^2 \sin^2 \gamma}}, \quad (19)$$

Eq. (3) is transformed to the expression, convenient for a construction of a numerical scheme:

$$\begin{aligned} q_r(r) &= 2\pi \int_0^\infty \int_0^{\pi/2} \cos \gamma \sin \gamma \cdot \left[ I_{\omega,R} \exp(\tau_R^r) \right. \\ &\quad + \int_r^R n_\omega^2 I_{0\omega} \exp(\tau_r^{r'}) d\tau_r^{r'} - I_{\omega,R} \exp(\tau_R^{r \sin \gamma} + \tau_r^{\sin \gamma}) \\ &\quad - \int_{r \sin \gamma}^r n_\omega^2 I_{0\omega} \exp(\tau_r^{\sin \gamma} + \tau_r^{r \sin \gamma}) d\tau_r^{r'} \\ &\quad \left. - \int_{r \sin \gamma}^r n_\omega^2 I_{0\omega} \exp(\tau_r^{r'}) d\tau_r^{r'} \right] d\gamma \, d\omega. \end{aligned} \quad (20)$$

Here  $R \sin \beta = r \sin \gamma$  and  $\gamma = \pi - \varphi$ . The use of a subsidiary function [10]:

$$\Theta(r, \tau) = r \left[ T(r, \tau) - T_R(\tau) \frac{T_0(r)}{T_{R,0}} \right], \quad (21)$$

allows transforming Eq. (12) to the form of a transfer problem, known as the Dirichlet problem:

$$\frac{\partial \Theta(r, \tau)}{\partial \tau} = a_1^2 \frac{\partial^2 \Theta(r, \tau)}{\partial r^2} + r \cdot f(r, \tau). \quad (22)$$

Here  $a_1^2 \equiv \frac{\lambda_1}{\rho_1 c_{p,1}}$ . The source function is unambiguously defined by local radiant flux in a droplet:

$$f(r, \tau) = \frac{1}{\rho_1 c_{p,1} r^2} \frac{\partial}{\partial r} [r^2 q_r(r, \tau)] + \frac{a}{r^2} \frac{T_R(\tau)}{T_{R,0}} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial T_0(r)}{\partial r} \right] - \frac{T_0(r)}{T_{R,0}} \frac{dT_R(\tau)}{dt}. \quad (23)$$

Noticing that the peculiarities of the function (21) ensure uniform conditions of unambiguosity

$$\Theta(0, \tau) = 0, \quad \Theta(R, \tau) = 0, \quad \Theta(r, 0) = 0 \quad (24)$$

and taking into account that the source function (23) in iterative calculations can be calculated according to the temperature field, determined in earlier iteration, Eq. (12) is transformed to integral form:

$$T(r, \tau) = T_R(\tau) \frac{T_0(r)}{T_{R,0}} + \frac{2R}{r} \sum_{n=1}^{\infty} \sin \frac{n\pi r}{R} \times \int_0^{\tau} \left\{ \frac{(-1)^n}{n\pi} \frac{T_0(r)}{T_{R,0}} \frac{dT_R(\tau')}{d\tau'} - \int_0^R \left( \frac{n\pi r'}{R} \cos \frac{n\pi r'}{R} - \sin \frac{n\pi r'}{R} \right) \left[ \frac{a_1}{R^2} \frac{T_R(\tau')}{T_{R,0}} \frac{dT_0(r')}{dr'} + \frac{1}{R^2 \rho_1 c_{p,1}} q_r(r', \tau') \right] dr' \right\} \exp \left[ -a_1 \left( \frac{n\pi}{R} \right)^2 (\tau - \tau') \right] d\tau'. \quad (25)$$

The solution algorithm of the integral equations (20) and (25) is formed by predicting control time  $\tau_I$  and by dividing it into  $I - 1$  number of finite size time intervals; a droplet radius is divided into  $J - 1$  number of finite size intervals of the droplet coordinate:

$$\tau_I = \sum_{i=2}^I \Delta \tau_i = \sum_{i=2}^I (\tau_i - \tau_{i-1}), \quad (26)$$

$$R_J = \sum_{j=2}^J \Delta r_j = \sum_{j=2}^J (r_j - r_{j-1}). \quad (27)$$

Then time integral in Eq. (25) and droplet coordinate integrals in Eqs. (20) and (25) are changed by the sum of integrals in the above-mentioned intervals and solved analytically.

Temperature of an evaporating droplet surface for time  $\tau_i$  is calculated using the fastest descent method, minimizing the numerical expression of the condition (4):

$$Nu_I \frac{\lambda_{v,g,I} (T_g - T_{R,I})}{2R_0} \int_{B,I}^{f,-0.7} + \frac{D_I}{T_{R,I}} \frac{L_I \mu_{v,I}}{R_0} p \ln \frac{p - p_{v,\infty,I}}{p - p_{v,R,I}} - \lambda_{l,I,J} \frac{T_{R,I}}{T_{R,0}} \frac{dT_0}{dr} \Big|_{I,J} - \frac{2\pi}{R_0^2} \lambda_{l,I} \sum_{n=1}^{\infty} (-1)^n n \sum_{i=2}^I \frac{1}{a_{i-1}} \left( \frac{R_0}{n\pi} \right)^2 \times \left[ \frac{(-1)^n}{n\pi} R_0 \frac{dT_R}{d\tau} \Big|_i + \frac{E_i}{R_0 \rho_{L,i} c_{p,L,i}} \right] \left\{ \exp \left[ a_{i-1} \left( \frac{n\pi}{R_0} \right)^2 (\tau_i - \tau_I) \right] - \exp \left[ a_{i-1} \left( \frac{n\pi}{R_0} \right)^2 (\tau_{i-1} - \tau_I) \right] \right\} = 0. \quad (28)$$

Temperature  $T_{R,I}$ , which satisfies the condition

$$\left( 1 - \frac{q_{\Sigma,i}^+}{q_{\Sigma,i}^- + q_{f,i}^+} \right) \cdot 100\% < 0.01\% \quad (29)$$

is found.

Time intervals  $\Delta \tau_i$  are selected in such way that during these intervals it would be possible to neglect the influence of an evaporating droplet dimension change on transfer process interaction. The parameter  $E_i$  evaluates the peculiarities of radiant absorption and the initial temperature field of a droplet. This parameter is calculated by modifying the expression, presented in the study [4], assuming that the change of the gradient of the droplet initial temperature field is linear in  $\Delta r_j$ . Local radiant heat flux in a droplet is calculated, by dividing radiant spectrum  $1.25 \times 10^6 \div \omega_2$  uniformly into 150 parts and integrating by rectangular method. The upper boundary of radiant spectrum interval  $\omega_2$  is selected according to the droplet size. Integration according to the angle  $\gamma$  is performed according to the Gauss method. Light beam reflection on liquid surface, Brewster's angle and the influence of warming droplet temperature on water spectral complex index of refraction are evaluated [30,31]. As the droplet surface temperature is determined, other parameters of heat and mass transfer are calculated. The droplet diameter is corrected:

$$R_I^3 = R_0^3 - 3 \sum_{i=2}^I \left[ \frac{m_i^+ + m_{i-1}^+}{\rho_{1,m,i} + \rho_{1,m,i-1}} R_0^2 + \frac{R_0^3}{1.5(\rho_{1,m,i} + \rho_{1,m,i-1})} \frac{d\rho_{1,m}}{d\tau} \Big|_i \right] \Delta \tau_i. \quad (30)$$

Then the boundary conditions (17) are corrected and the aforementioned calculations are repeated until the condition  $Kn < 0.3$  is fulfilled.

### 3. Results and discussion

As droplets warm and evaporate, the boundary conditions of heat and mass transfer in a two-phase flow vary. Quantity of sprayed liquid, defined by the ratio of gas and liquid initial flow rates  $g_0 = G_{1,0}/G_{g,0}$ , decides the change of the gas flow state. When this parameter is close to zero, warming and evaporation of liquid does not affect gas state. We have a single droplet case. This case is important, because it allows highlighting the peculiarities of transfer processes in a droplet, eliminating the influence of the change of the gas flows state.

The initial velocity of droplet slip in gas  $\Delta w_0 = w_{1,0} - w_{g,0}$  and the intensity of radiant flux absorption determine the mode of droplet heating. Mostly often the initial velocity of droplet is not equal to the gas velocity. Therefore droplets slip in gas. Though under the influence of friction drag forces the velocity of droplet slip in gas decreases and radiant-convective heating of droplets gradually turns into radiant-conductive. As liquid intensively evaporates and the droplets become smaller, radiant absorption in the

droplets decreases, meanwhile conductive heating intensity increases. Therefore in the final stage of droplet evaporation the conditions of conductive heating prevail. In order to evaluate the peculiarities of combined droplet heating, it is necessary to know the change of the droplet state in the case of the simplest boundary conditions.

3.1. Peculiarities of the state change of an evaporating droplet in the conductive heating case

The change of the droplet state in the conductive heating case is thoroughly examined, by modelling heating and evaporation of water droplets in air at temperature  $T_d = 1273$  K, as droplet slip velocity equals to zero. It is determined, that in the case of conductive heating without reference to the initial water temperature and the size of droplets, during unsteady evaporation the droplets reach a new isothermal state (Fig. 1a). The droplet surface temperature  $T_{R,e,k}$  can determine this state. The droplet warms up to this temperature at the moment  $\tau_e$  of equilibrium evaporation beginning. The temperature field gradient of the conductively heated droplet continuously decreases (Fig. 1b) and it becomes equal to zero, as it reaches equilibrium evaporation mode. The temperature field gradient is

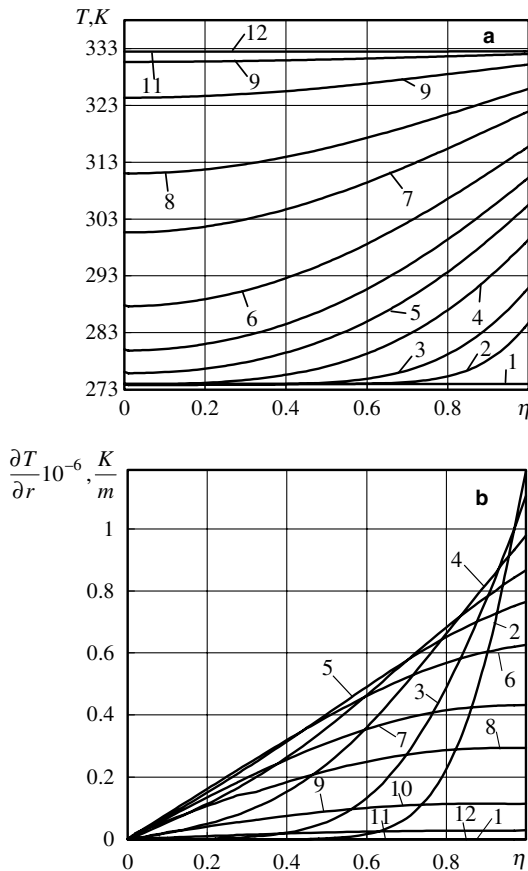


Fig. 1. Temperature field (a) and its gradient (b) in a water droplet in the case of conductive heating.  $\tau$  (s): (1) 0, (2) 0.0006, (3) 0.0012, (4) 0.0023, (5) 0.0035, (6) 0.0047, (7) 0.0064, (8) 0.0093, (9) 0.012, (10) 0.019, (11) 0.27, (12) 0.055;  $R_0 = 75 \times 10^{-6}$  m;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ .

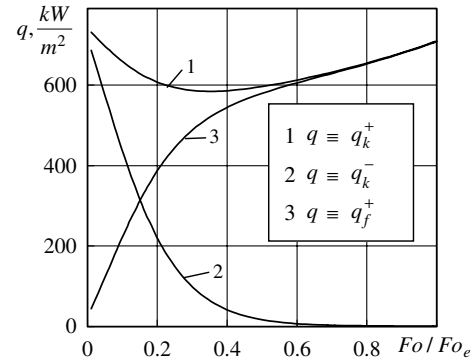


Fig. 2. Variation of heat fluxes on an evaporating droplet surface in the case of conductive heating.  $R_0 = 10^{-4}$  m;  $T_{R,0} = 303$  K;  $p_{v,\infty}/p = 0.1$ .

directed towards the droplet surface; its modulus corresponds to the part of external conductive heat flux, which heats the droplet. The intensity of the droplet heating during its unsteady evaporation is defined by conductive heat flux on the internal side of the droplet surface:  $q_h \equiv q_k^-$ . The intensity of heat input for phase transformations is indicated by the difference of conductive heat fluxes on the internal and external sides of the droplet surface:  $q_f \equiv q_k^+ - q_k^-$ . During unsteady evaporation  $q_h$  gradually decreases, meanwhile  $q_f$  increases (Fig. 2). As equilibrium evaporation mode is reached,  $q_f \equiv q_k^+$ . Generalizing research results from various cases allows to state, that during equilibrium evaporation droplet temperature remains constant and the droplet remains isothermal till its extinction moment only in the case of its conductive heating, if the change of the droplet state has no influence on the parameters of gas flow. Hence, the condition  $T_{R,e,k}(\tau) = \text{const}$  is the peculiarity of a single droplet conductive heating.

The dynamics of the droplets that differ in size and have different initial liquid temperature is distinctive. That is shown by the particular dynamics of their surface temperature variation (Fig. 3). Bigger is the difference between the ratio of temperatures  $T_{R,0}/T_{R,e,k}$  and 1; bigger is divergence of droplet surface temperature curves, as dispersivity of sprayed liquid increases. Though the droplets reach equilibrium evaporation mode being at the same energetic level. Hence, as the droplet is in its equilibrium evaporation mode and is heated by conduction, its surface temperature value does not depend on the droplet size and the initial state.  $T_{R,e,k}$  corresponds to the smallest droplet surface temperature, at which an equilibrium droplet evaporation can occur at the given parameters  $T_g$  and  $p_{v,\infty}$ . Temperature of the droplet during its equilibrium evaporation depends on gas temperature and is sensible to the pressure ratio  $p_{v,\infty}/p$ .

The dynamics of heating of evaporating droplets, represented in universal time scale, does not depend on the droplet size (Fig. 4a). That is very important. Though during unsteady evaporation of the droplet the curves of  $T_R(\tau)$  variation stratify subject to dispersed water temperature.

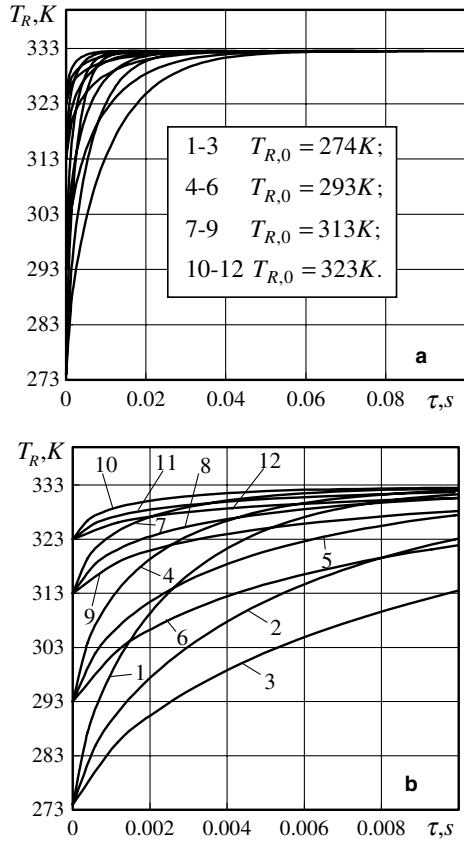


Fig. 3. Variation of droplet surface temperature during its unsteady evaporation.  $R_0 \times 10^6$  (m): (1, 4, 7, 10) 50, (2, 5, 8, 11) 75, (3, 6, 9, 12) 100;  $p_{v,\infty}/p = 0$ ;  $\Delta w = 0$ .

Every curve represents heating dynamics of large droplets of different size. More thorough analysis of the obtained results shows that the dynamics of non-dimensional temperature

$$T_{R,n} = \frac{T_R - T_{R,0}}{T_{R,e,k} - T_{R,0}} \quad (31)$$

of heating and evaporating droplets is universal (Fig. 4b). Determined peculiarity of the droplet state change in the case of conductive heating is important when creating the method of engineering calculation of unsteady droplet evaporation. The effect of heating droplets that have different initial temperature evokes different growth of the droplet radius (Fig. 5a). The influence of the droplet expansion effect on its radius dynamics is more significant, as the difference between droplet temperature and equilibrium evaporation temperature  $T_{R,e,k}$  is bigger. The curves of mass flux on the surface of the similar size droplets also stratify subject to the difference between initial liquid temperature  $T_{R,0}$  and equilibrium evaporation temperature  $T_{R,e,k}$ . Larger are the droplets; higher is the influence of the initial liquid temperature at the initial stage of the heating period (Fig. 5b). The curves of variation of non-dimensional vapour flux (Fig. 6a) and non-dimensional droplet radius (Fig. 6b) in the universal time scale do not depend on the

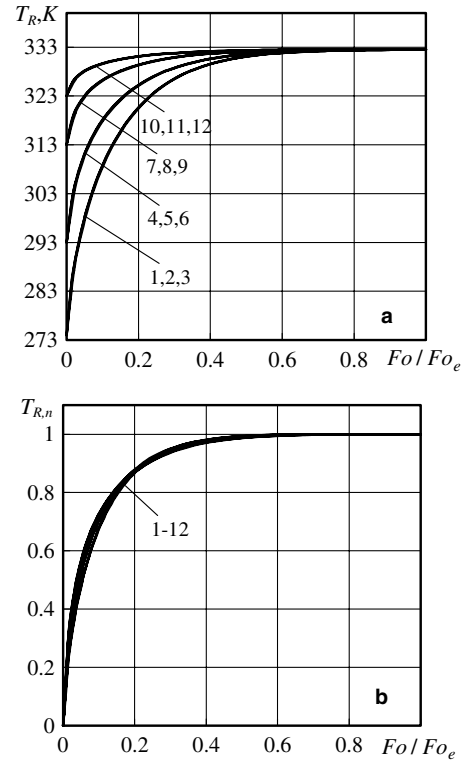


Fig. 4. Change of non-dimensional temperature of a water droplet surface during unsteady evaporation in the universal time scale. Boundary conditions the same as in Fig. 3.

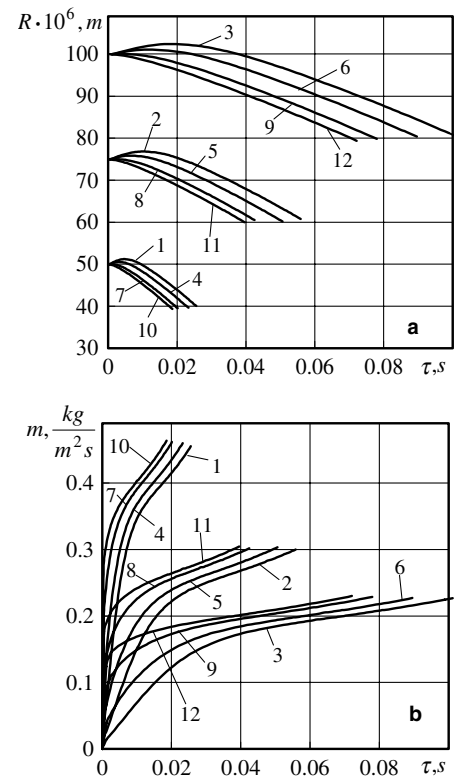


Fig. 5. Dynamics of unsteady evaporation of water droplets. Boundary conditions the same as in Fig. 3.



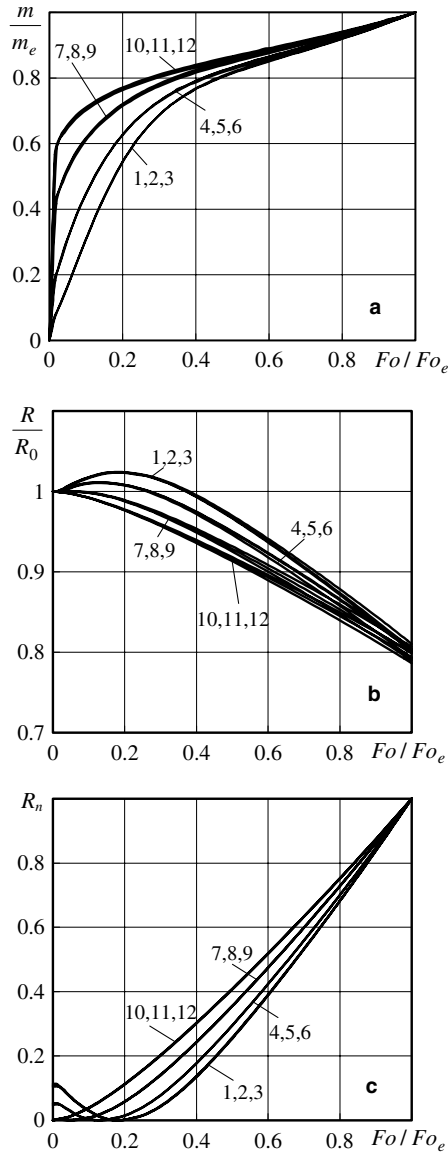


Fig. 6. Dynamics of unsteady evaporation of water droplets in the universal time scale. Boundary conditions the same as in Fig. 3.  $R_n = (R_{max} - R)/(R_{max} - R_0)$ .

droplet dimension at the same initial temperature of liquid. Though the attempts to obtain the curve of universal evaporation dynamics in the case of various initial temperatures of liquid were not successful due to the peculiar effect of a droplet volume expansion (Fig. 6c). It is obvious, that the method of engineering calculation of unsteady droplet evaporation can be constructed only according to the dynamics of variation of the universal droplet non-dimensional temperature. It is necessary to take into account the value of liquid unsteady evaporation temperature in the case of conductive heating of droplets. This temperature will depend on gas temperature (Fig. 7) and on liquid nature. In the case of water evaporation, air humidity influence is important (Fig. 8). Furthermore, the influence of complicated boundary conditions of heat and mass transfer

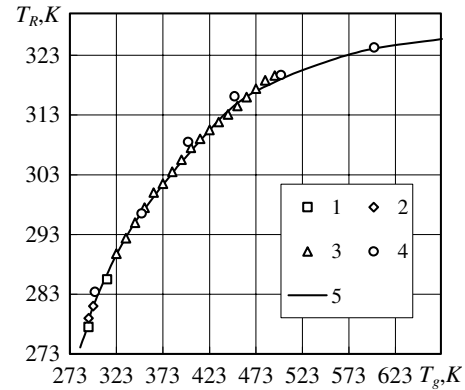


Fig. 7. Temperature of droplets that evaporate in equilibrium mode in dry air. Points—experiment [5,32]. Lines—numerical research in the case of conductive heating.

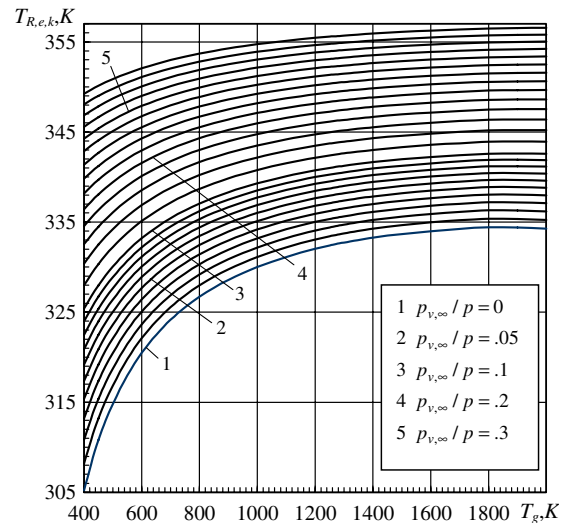


Fig. 8. Dependence of temperature of droplets, evaporating in equilibrium mode on air parameters. Conductive heating case.

on the evaporating droplet temperature should be evaluated.

### 3.2. The state change of an evaporating droplet in combined heating cases

The cases of radiant–conductive, convective and radiant–convective heating of water droplet are examined. Convective heating is modelled assuming that the initial velocity of droplet slip in gas  $\Delta w = 10$  m/s. Radiant–conductive and radiant–convective heating cases are modelled assuming that an additional black source of radiation exists, temperature of which is equal to air temperature.

The temperature mode of a heating droplet essentially changes in the case of radiant–conductive heating case (Fig. 9). The initial (Fig. 9a) and the transient (Fig. 9b) periods of the droplet state change distinguish quite significantly during the unsteady evaporation mode. During the initial heating period the temperature field of the droplet

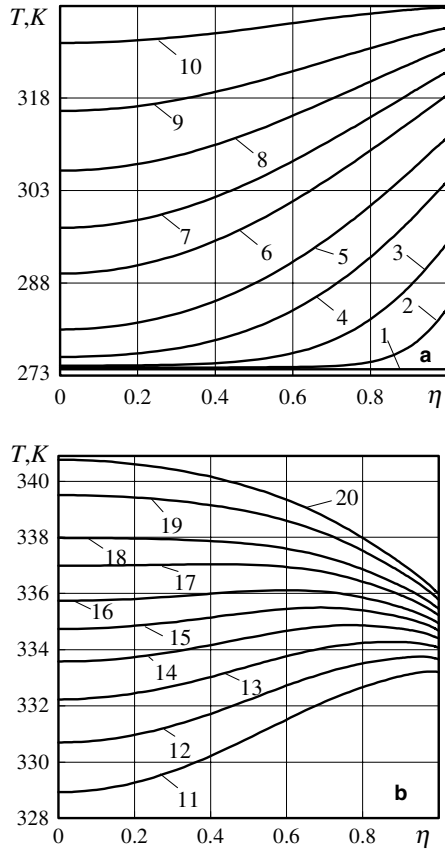


Fig. 9. Temperature field of a water droplet during its unsteady evaporation: (a) the initial heating period; (b) the transient heating period. Time of radiant-conductive heating  $\tau$  (s): (1) 0, (2) 0.0008, (3) 0.0024, (4) 0.0041, (5) 0.007, (6) 0.01, (7) 0.0124, (8) 0.0155, (9) 0.0202, (10) 0.0256, (11) 0.0271, (12) 0.0287, (13) 0.0302, (14) 0.0318, (15) 0.0333, (16) 0.0349, (17) 0.0372, (18) 0.0396, (19) 0.045, (20) 0.058;  $R_0 = 10^{-4}$  m;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ .

qualitatively corresponds to the conductive heating case: maximal temperature is on the droplet surface; minimal temperature—in the droplet centre. During the transient period temperature maximum is shifted inside the droplet. During the initial stage of this period maximal temperature gradually shifts to the droplet centre and stays there, though the droplet warms further and reaches the conditions that correspond to equilibrium evaporation mode. The droplet reaches the equilibrium evaporation mode being maximally heated up. The change of the temperature field gradient (Fig. 10) and the diagram of energy fluxes on the droplet surface (Fig. 11) indicate the peculiarities of the droplet heating during unsteady evaporation mode. It can be seen that in the case of radiant-conductive heating the temperature field gradient undergoes not only qualitative, but also quantitative transformations: the direction and magnitude of its vector change. Therefore the meaning of the conductive heat flux  $\vec{q}_k$  on the internal side of the droplet surface changes as well. At the initial heating period the vector of  $\vec{q}_k$  is directed towards the droplet centre; its magnitude corresponds to the part of the external conductive heat flux  $q_k^+$ , which heats the droplet. On the whole, the

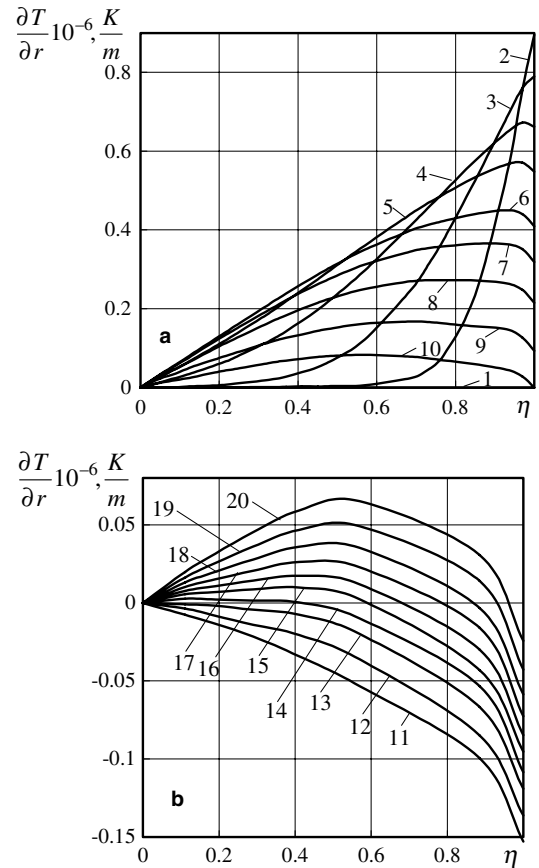


Fig. 10. Temperature field gradient of a water droplet in the case of radiant-conductive heating. Boundary conditions the same as in Fig. 9.

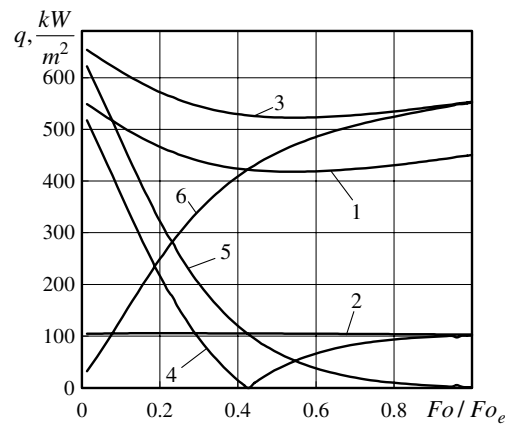


Fig. 11. Variation of energy fluxes on a droplet surface during its unsteady evaporation. Meaning of the parameter  $q$ : (1)  $q \equiv q_k^+$ ; (2)  $q \equiv q_r^-$ ; (3)  $q \equiv q_k^-$ ; (4)  $q \equiv q_r^+$ ; (5)  $q \equiv q_k^+$ ; (6)  $q \equiv q_r^-$ ;  $R_0 = 10^{-4}$  m;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ .

droplet is additionally heated by entire radiant energy, absorbed in it:  $q_h \equiv q_k^- + q_r^-$ . When the transient heating period begins, temperature maximum is shifted towards the droplet centre and the direction of the gradient changes; only radiant energy can pass into the droplet. Though part of this energy is already taken into the droplet

surface by conduction and participates in evaporation process:  $q_f = q_k^+ + q_k^-$ . Hence the vector of conductive heat flux  $\vec{q}_k^-$  is directed towards the droplet surface; its magnitude corresponds to the part of absorbed radiant flux, which evaporates the droplet. Droplet heating intensity is determined by the total heat flux  $q_\Sigma = q_r - q_k^-$ . As equilibrium evaporation mode approaches, its value becomes equal to zero (Fig. 11, curve 5). The intensity of heat supplied to the droplet is defined by the curve 3 of the total heat flux. As equilibrium evaporation begins, the curve 3 joins with the curve 6, which defines energy consumption for phase transformations;  $q_k^-$  curve 4 joins with the curve 2, which defines the intensity of radiant absorption in the droplet (Fig. 11). That shows that from this moment all radiant heat is involved in the droplet evaporation process. Droplet evaporation rates, calculated in the case of radiant–conductive heating of water droplets, correlate with the results of the experimental research of water droplet evaporation, presented in the study [28] (Fig. 12).

In the case of radiant–conductive heating a droplet reaches its equilibrium evaporation mode being non-isothermal. It is the peculiarity of this case. Non-isothermality is quite significant. It is shown by the curves 4, 5 and 6 of the droplet surface, centre and mean mass temperatures (Fig. 13). The increment of the droplet surface temperature, if compared to a droplet surface temperature in the case of conductive heating, is decided by the ratio of radiant and conductive components of the total external heat flux (Fig. 14):

$$\Delta T_{e,r-k} = T_{R,e,r-k} - T_{R,e,k} = f(q_{r,e}^+/q_{k,e}^+) \quad (32)$$

In the case of convective heating droplet temperature increment from the conductive heating case is more significant, though non-isothermality in the droplet is not so distinct (Fig. 13, curves 7–9). That is decided by the significant influence of circulation on heat transfer in the droplet (Fig. 15, curve 1), which gradually decreases, as maximal velocity of liquid flow on the droplet surface (Fig. 15, curve 3) and Peclet number decrease. Maximal velocity of liquid

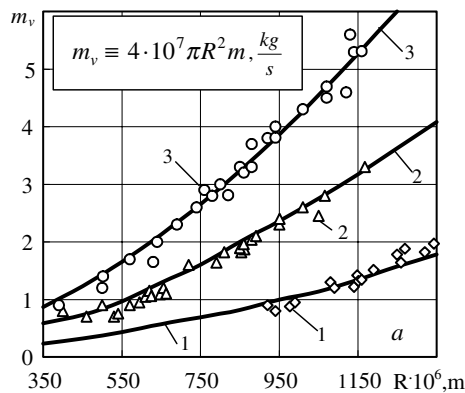


Fig. 12. The rate of water droplet evaporation in the case of radiant–conductive heating. Points—experiment [28]. Lines—numerical research.  $T_g$  (K): (1) 678; (2) 889; (3) 1043.

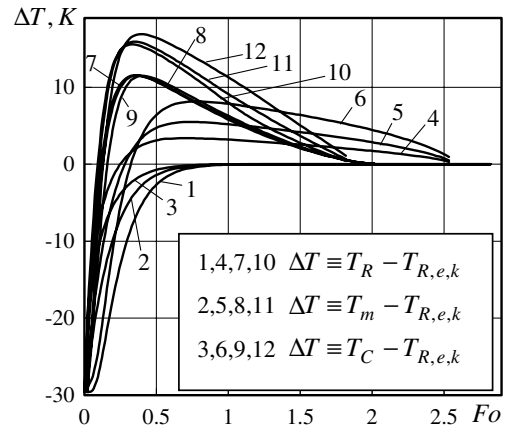


Fig. 13. Dependence of droplet non-isothermality on its heating manner: (1–3) conductive, (4–6) radiant–conductive, (7–9) convective; (10–12) radiant–convective;  $R_0 = 10^{-4}$  m;  $T_{R,0} = 303$  K,  $p_{v,\infty}/p = 0$ .

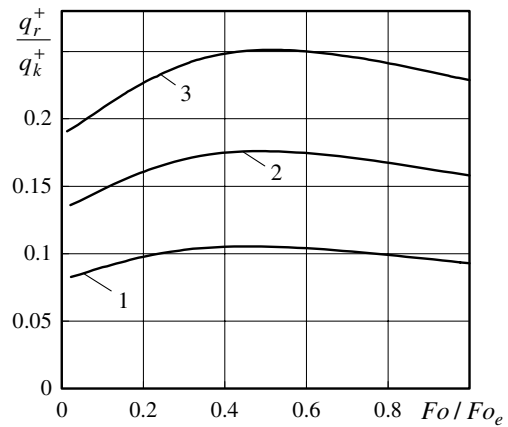


Fig. 14. Variation of the components of the total heat flux.  $R_0 \times 10^6$  (m): (1) 50; (2) 75; (3) 100;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ .

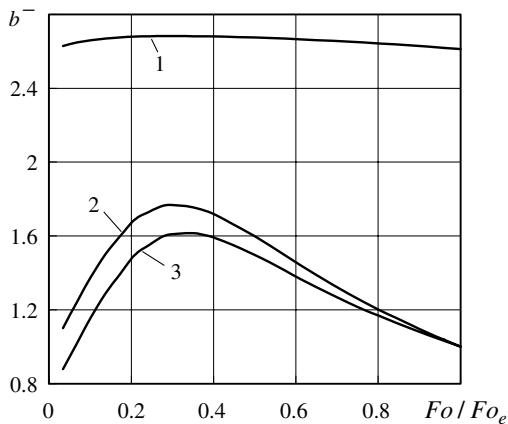


Fig. 15. Variation of the parameters that determine the intensity of water circulation during unsteady evaporation.  $\Delta w_0 = 10$  m/s;  $R_0 = 10^{-4}$  m;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ ; Meaning of the parameter  $b^-$ : (1)  $b^- \equiv k_c$ , (2)  $b^- \equiv Pe_f/Pe_{1,e}$ , (3)  $b^- \equiv w_l/w_{1,e}$ .

flow on the droplet surface and Peclet number for liquid are calculated according the method presented in [7]. By the end of unsteady evaporation the influence of liquid circulation on heat transfer in water droplets is still significant. As the velocity of droplet slip in gas decreases during equilibrium evaporation, this influence gradually weakens and disappears as the convective heating mode approaches to the conductive one. Only qualitative changes of the evaporating droplet state change can be observed, if compared to the conductive heating case. In the case of convective heating, similarly to the conductive heating case, the droplet reaches its equilibrium evaporation mode being isothermal, though at higher energetic level. The increment of the droplet surface temperature, if compared to the droplet surface temperature in the case of conductive heating, is decided by the ratio of convective and conductive heat fluxes:

$$\Delta T_{e,c} = T_{R,e,c} - T_{R,e,k} = f(q_{c,e}^+/q_{k,e}^+). \quad (33)$$

As the velocity of droplet slip in gas decreases,  $\Delta T_{e,c} \rightarrow 0$ . During unsteady evaporation the gradient of temperature field in the droplet is positive;  $q_k^-$  corresponds to the part of the total external heat flux  $q_{\Sigma}^+ = q_k^+ + q_c^+$  that heats the droplet. The droplet cools during its equilibrium evaporation; that is conditioned by decreasing velocity of the droplet slip in gas due to friction drag forces. Convective heating of the droplet gradually turns into conductive one and its temperature gradually approaches to temperature of droplet equilibrium evaporation in the case of conductive heating  $T_{R,e,k}$ .

In the case of radiant–convective heating the droplet warms up to the highest temperature (Fig. 13, curves 10–12). Its non-isothermality is more distinct if compared to the convective heating case, though less distinct if compared to the radiant–conductive one. The parameters, which determine the intensity of the heat flux supplied to the droplet from outside, vary during unsteady evaporation (Fig. 16). In this case the increase of the ratios of conductive on convective component (Fig. 16, curve 1) and radiant on convective component (Fig. 16, curve 2) of the total external heat flux is decided by (1) the decreasing velocity of droplet slip in gas (Fig. 16, curve 3); (2) rapid warm of dispersed liquid and (3) the increasing intensity of phase transformations. As the slip velocity and the difference between gas and droplet temperatures decrease and the influence of the Stefan hydrodynamic flow increases (Fig. 16, curve 5), the convective heat flux decreases for some period of time. After that it gradually increases due to growing influence of decreasing droplet diameter. Heat flux of absorbed radiation is decided by temperature of external radiation source and by the droplet size. During the period of intensive droplet heating evaporation rate is not high. Besides, the influence of phase transformations on the decreasing droplet dimension is partly compensated by the effect of volumetric expansion; therefore the radiant flux varies insignificantly during unsteady evaporation. In the case of radiant–convective heating the increment of

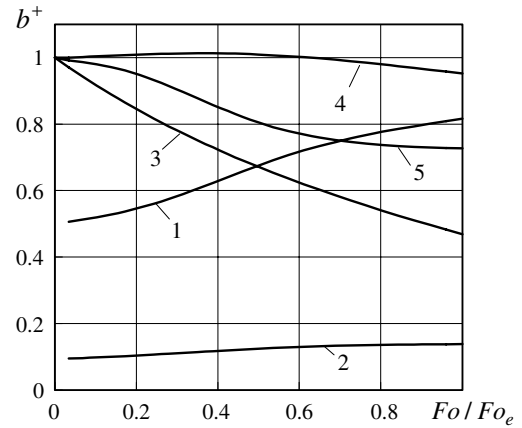


Fig. 16. Variation of the parameters that determine external heating intensity during unsteady evaporation.  $\Delta w_0 = 10$  m/s;  $R_0 = 10^{-4}$  m;  $T_{R,0} = 274$  K;  $p_{v,\infty}/p = 0$ ; Meaning of the parameter  $b^+$ : (1)  $b^+ \equiv q_k^+/q_c^+$ , (2)  $b^+ \equiv q_r^+/q_c^+$ , (3)  $b^+ \equiv \Delta w/\Delta w_0$ , (4)  $b^+ \equiv R/R_0$ , (5)  $b^+ \equiv Nu_v/Nu$ .

the droplet surface temperature, if compared to equilibrium evaporation temperature of the droplet surface in the case of conductive heating, can be defined by the function of the ratios  $q_{r,c}^+/q_{c,e}^+$  and  $q_{c,e}^+/q_{k,e}^+$ :

$$\Delta T_{e,r-c} = T_{R,e,r-c} - T_{R,e,k} = f(q_{c,c}^+/q_{k,e}^+; q_{r,c}^+/q_{c,e}^+). \quad (34)$$

The interaction of radiation and thermal conduction processes in semitransparent liquid makes the essential influence on the state of an evaporating droplet. This interaction deforms unsteady temperature field and alters heat fluxes. Interaction intensity can be evaluated by the ratios of the components of the total external heat flux  $q_{\Sigma}^+$ : in the case of radiant–conductive heating:  $q_r^+/q_k^+$ ; in the case of radiant–convective heating:  $q_r^+/q_c^+$ . Values of the ratio

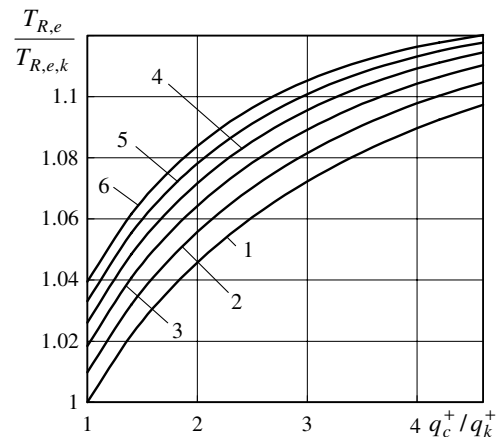


Fig. 17. The influence of boundary conditions on water droplet surface temperature at the moment of unsteady evaporation beginning. The ratio of radiant and convective heat fluxes  $q_r^+/q_c^+$ : (1) 0; (2) 0.2; (3) 0.4, (4) 0.6, (5) 0.8, (6) 1.

$q_c^+/q_k^+$  and of the above-mentioned ratios determine temperature of a water droplet surface at the moment of equilibrium evaporation beginning (Fig. 17).

#### 4. Conclusion

The regularities of the state change of water droplets, evaporating under various conditions of heat and mass transfer, are determined in this study. These regularities make assumptions to construct the method of engineering calculation of transfer processes in two-phase gas and droplet flows. It is obvious that the methodology of such method can rely on the dynamics of variation of the normalized heating water droplet surface temperature in the universal time scale  $Fo/Fo_e$  in the case of conductive heating. Stating that normalized temperature  $T_{R,n}$  of the water droplet surface is known in the universal time interval between 0 and 1, temperature of the water droplet surface  $T_{R,e,k}$  at the moment of equilibrium evaporation beginning (as  $Fo/Fo_e = 1$ —Fig. 8) in the case of conductive heating is determined according to gas temperature  $T_g$  and water vapour partial pressure in gas  $p_{v,\infty}/p$ . Then, taking count of the initial droplet size, the ratios of the heat fluxes  $q_c^+/q_k^+$  and  $q_r^+/q_c^+$  are approximately calculated; the increment of the droplet surface temperature  $\Delta T_e$  in the case of combined droplet heating (Fig. 17) is determined. The convective heat flux into the evaporating droplet is calculated according to the expression (9). The conductive heat flux is also calculated according to the expression (9), as  $Nu = 2$ . The heat flux of radiation, absorbed in the droplet, is calculated according to the Stefan–Boltzman law, evaluating the effective absorption factor [11]. Then instant temperatures of the droplet surface are calculated:

$$T_{R,i} = T_{R,0} + T_{R,n,i}(T_{R,e} - T_{R,0}). \quad (35)$$

The vapour fluxes  $m_i$  are calculated according to the expression (3). The first-approach radius  $R_e$  for the instant of equilibrium evaporation beginning is determined by the expression (30). Then it is possible to correct the ratios  $q_c^+/q_k^+$  and  $q_r^+/q_c^+$ , temperature  $T_{R,e}$  and to calculate the corrected droplet radius  $R_e$ . During the equilibrium evaporation mode similar calculations are performed, freely selecting intermediate control values of the droplet radius between 0 and  $R_e$ , though evaluating that in the conditions of equilibrium evaporation  $T_{R,e,k}(\tau) = \text{const}$  for a single droplet.

Such method of engineering calculation of the droplet problem does not require a lot of CPU time. This method is quite convenient for engineering evaluation of the intensity of combined transfer process interaction and opens wide possibilities not only for practical application of the droplet problem, but also for the use of the method in CFD (Computational Fluid Dynamics) codes. For that it would be convenient to have the disposition of empirical expressions that describe the dynamics of temperature  $T_{R,n}$  in the wide range of boundary conditions, also

expanding spectrum of various semitransparent liquids. That is the goal of further research.

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